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QUANTUM HALL EFFECT: THE FUNDAMENTALS

The basic understanding of the physics behind and the reasons for very high precision of the resistivity ρ_{xy} quantisation in integer quantum Hall effect (IQHE) and the application of the effect in metrology to define a quantum resistance standard will be briefly discussed. We also mention some recent proposals concerning the application of the quantum Hall device as an efficient qubit for future quantum computers and end up with few remarks about the contribution of single electron devices to the realisation of standards and quantum metrology which seeks the ways to beat the accuracy of classic measurements.

Keywords: quantum Hall effect, resistance standard, quantum metrology, precision measurements

1. INTRODUCTION: FROM EDWIN HERBERT HALL TO KLAUS VON KLITZING

Electrons in bulk system can move freely in all three dimensions and often can be treated as free quantum quasi-particles[1]. This is no longer the case in modern structures in which their motion is restricted to two dimensions (2D *i.e.* a plane), one dimension (1D - the quantum wire) or even zero dimensions (0D - quantum dot). In such structures the electrons display very unusual behaviour. New quantum liquids with properties much different from the well known Fermi liquid are possible. These include variety of quantum Hall liquids and composite fermion states in 2D, Luttinger liquid in 1D, etc. The charge of elementary excitations may amount to the fraction of electron charge and their statistical properties may dramatically differ from that of bosons and fermions in three dimensional systems [1].

In this work we shall concentrate on the magnetotransport phenomena observed in 2D electron gas. Let us start the discussion with a three dimensional system in the form of a thin conducting metallic or semiconducting slab. As it is well known the application of a magnetic field perpendicular to the surface of such a system along which the current flows produces a voltage V_H across the sample transverse to the current flow (see Fig. 1 for a typical setup). The effect has first been observed in 1879 by the graduate student of John Hopkins University, Edwin Hall[2] [2]. The appearance of voltage, V_H is known as the Hall effect. It is due to the Lorentz force acting on charges moving in the presence of the magnetic field. In the equilibrium the magnetic part qv_DB of the Lorentz force F_L is balanced by the electric one $qE_x = qV_H/L_y$, so $V_H/L_y = v_DB$. Here v_D is the average drift velocity of carriers, q – their charge and L_y – width of the sample. Noting that the current I_x can be expressed as the product of the drift velocity v_D, charge density n, and the cross sectional area of the sample $S = L_yw$ (where w is the thickness of the slab) we find the perpendicular resistivity $R_{yx} = V_H/I_x$ to be

[1] The term quasi-particle is used to denote the particle which properties i.e. effective mass differ from the original particle. For example the effective mass m_ of an electron in GaAs heterostructure is about 0.07 electron mass me.
[2] Edwin Herbert Hall born November 7, 1855, Great Falls (later North Gorham), Maine, U.S.A died November 20, 1938, Cambridge, Mass. U.S.A.

$$R_{yx} = \frac{B}{qnw} = \frac{B}{qN_S}, \tag{1}$$

where n is the number of carriers per unit volume. Note, that nw is the number of carriers per surface area, to be denoted N_S.

The measurement of the Hall resistance (or Hall constant $R_H = R_{yx}/B$) gives information about the density N_S and the sign ($q = \pm e$) of charge carriers in metals and semiconductors. The effect became a standard tool of material characterisation. The direct proportionality of the Hall resistivity to the local magnetic field B is used to measure the magnetic field and its distribution [3].

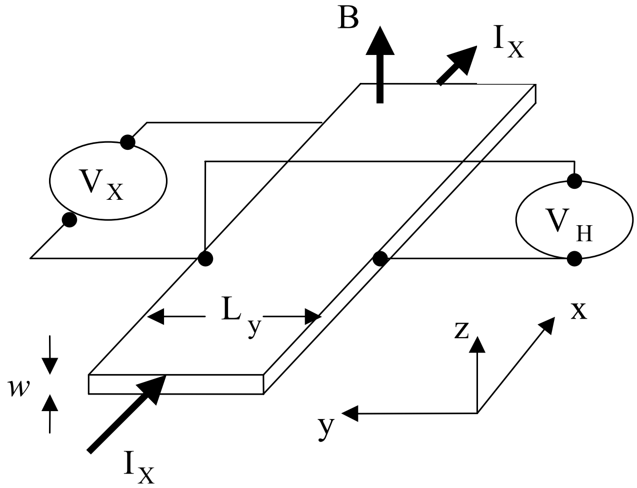

Fig. 1. The standard geometry used in the studies of the Hall effect in 3 dimensional samples and quantum Hall effects in 2 dimensional electron (or hole) gas.

All this concerns three dimensional samples. What about the two dimensional systems? At first sight nothing special can be inferred from Eq. (1). So it came about as a big surprise, when Klaus von Klitzing's[3] and his colleagues' [4] measurements completely analogous to those in the classic Hall effect albeit performed on a two dimensional electron gas have shown strong departures from the expected behaviour. To be more precise: the low temperature measurements [4] of Hall resistance on the high mobility two-dimensional electron gas in a strong magnetic field have revealed highly nonlinear behaviour of $R_{yx}(B)$ for fields high enough. It manifests itself as a series of plateaus. In the original 1980 measurements a Si MOSFET device has been used in which the plateaus as a function of carrier density have been observed. In a semiconductor heterostructure in which carrier concentration is kept constant the plateaus, extending over a range of magnetic fields are observed. To a very high accuracy and independently of the material details one measures on the plateau

$$R_{xy} = \frac{h}{ve^2} = \frac{25812.807\Omega}{v}, \tag{2}$$

where h is Planck's constant, e – electron charge and v is a number.

In the rest of the paper I will shortly discuss some reasons behind the high precision of resistance quantisation, theoretical understanding and universality of the effect (section 2), the application of IQHE as a resistance standard and recent proposals to use both IQHE and

[3] Klaus von Klitzing was born 28th June 1943 in Środa near Poznań.

fractional QHE as working quantum information processors (section 3). I end up with some comments on the general issues connected with quantum metrology.

2. THE QUANTUM HALL EFFECT

When discussing quantum Hall effects one has to distinguish integer QHE (IQHE) when ν in Eq. (2) is an integer 1, 2, 3 ... and fractional QHE when the measured Hall resistance corresponds to $\nu = p/q$ and p and q are relatively prime integers. The appearance of the plateau in R_{yx} is in both cases accompanied by the vanishment (at low enough temperatures) of the longitudinal resistance $R_{xx} = I_x/V_x$, where V_x is the voltage drop along the sample. Even though the experimental manifestations of both effects is to high extent the same, the physics behind them is very different. We shall not discuss the theories of fractional QHE here as it is beyond the scope of this presentation.

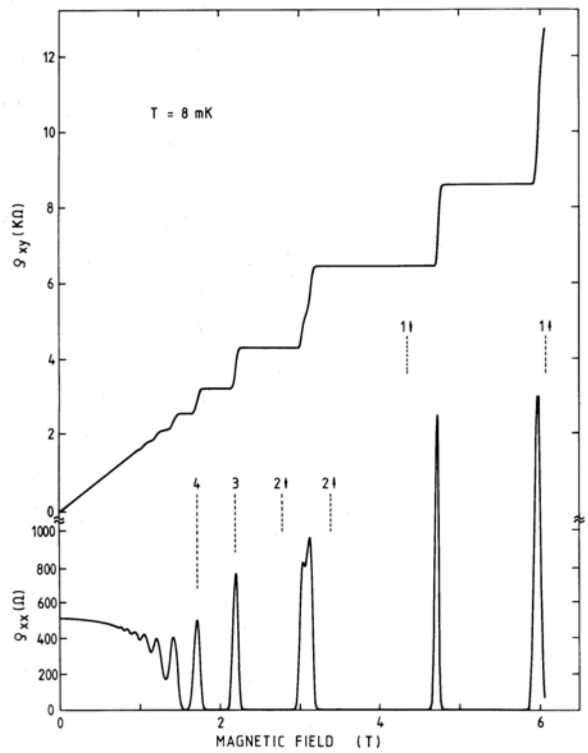

Fig. 2. The low temperature measurements of magnetotransport on the very high mobility 2DEG showing multitude of integer quantum Hall steps [6] (upper part) and the oscillations of the longitudinal resistance (lower part).

Figure (2) shows a series of well developed plateaus in the Hall resistance of the two dimensional electron gas formed in the high mobility GaAs/AlGaAs heterostructure. In the original paper [4] a relative accuracy 10^{-5} of quantisation was observed. At present the absolute precision of quantisation of R_{yx} is a few parts in 10^8 or better and the agreement between measurements on a different devices is few parts in 10^{10}. As a result the Hall resistance has been adopted as an international standard of resistance [5].

To understand the very possibility of the high precision of measurements it is important to realise that in two dimensions the Hall resistance R_{yx} and Hall resistivity ρ_{yx} coincide (notation as in Fig. (1))

$$R_{yx} = \frac{V_H}{I_x} = \frac{V_H / L_y}{I_x / L_y} = \frac{E_y}{j_x} = \rho_{yx} \,. \tag{3}$$

This means that single and potentially very accurate ”electric measurement” of R_{yx} suffices for the determination of resistivity ρ_{yx}. To get the information on the other component of the tensor i.e. the longitudinal resistivity ρ_{xx} both the knowledge of R_{xx} and sample dimensions L_x, L_y are necessary as $R_{xx} = V_x/I_x = E_xL_x/j_xL_y = \rho_{xx}L_x/L_y$. The measurements of L_x and L_y, however, are never very precise. Note, in two dimensions the resistance and resistivity are expressed in the same units, namely ohms.

In connection with the above statements recall that also the vanishment of the longitudinal resistance is very accurate. The value of it measured in the Hall plateau region is lower than in any non–superconducting material. Before we start with theoretical explanation of the IQHE we have to note another aspect connected with the precision of quantisation. Assuming typical electron concentration in 2DEG of the order of 2×10^{11} cm^{-2} and typical dimensions of the sample 260 μm × 400 μm [6] we find the number of electrons in a two dimensional channel $N \approx 2\times10^{8}$. Thus the precision of quantisation is of order $1/N$ instead of expected, on statistical grounds, much lower precision $1/\sqrt{N} \approx 10^{-4}$ connected with fluctuations of physical parameters in the many body system. These measurements thus provide an example of quantum measurements which beat the shot noise precision $1/\sqrt{N}$ (see discussion in section 3).

This fact poses a severe constraint on the acceptable theoretical explanation of the effect (which should not make use of, our favourite, statistical methods). There is a number of theoretical proposals to understand the independence of the result on material properties, geometry of the sample and other real life disturbances. The most general one has been proposed by Laughlin [7] and makes use of the gauge freedom. Other approaches invoke the scattering theory in impure 2D electron gas subject to a perpendicular magnetic field, edge states [8], etc. There exists a vast original literature on the subject [9, 10, 11], and a number of books [12].

As the simplest way of presenting the main ideas let us start with free 2D electrons with effective mass m^* and the spectrum $E(k_x, k_y) = (\hbar^2/2m^*)(k_x^2 + k_y^2)$. Under the action of a magnetic field the spectrum becomes completely quantized in highly degenerate Landau levels of energy $E_n = (n+1/2)\hbar\omega_c$, where $\omega_c = eB/m^*$ is the cyclotron energy. The degeneracy g of each of the levels $n = 0, 1, 2 ...$, can be calculated as the number of non penetrating cyclotron orbits allowed in a sample of surface S, $g = S/2\pi l^2 = \phi/\phi_0$, where $l = \sqrt{\hbar / eB}$ is the magnetic length, ϕ the magnetic flux and $\phi_0 = h/e$ is the flux quantum. In GaAs based heterostructure and a field of about 10 T the magnetic length $l \approx 8$ nm, while the cyclotron energy is of the order of meV . The surface density of electrons entering the Eq. (1) is obtained by multiplying the degeneracy per area (= eB/h) with the number ν of Landau levels below the Fermi level: $N_S = \nu eB/h$. If this is plaggued in into (1) with $q = e$ one gets

$$R_{yx} = \frac{1}{\nu}\frac{h}{e^2} \,, \tag{4}$$

in numerical agreement with observation, but a completely wrong result. It merely says that for some precisely determined value of the magnetic field the value of the Hall resistance equals to the value given by the rhs. of the above equation. This does not explain neither the existence of the plateau for a range of magnetic fields nor the concommittal vanishing of the longitudinal resistance. In one way of thinking about the problem it is impurities which explain the physics behind the quantisation. In presence of impurities some of states become localised with energies lying in between Landau levels (see Fig. (3)). If the Fermi level lies in the region of localised states the current does not flow and the longitudinal resistance vanishes. An increase of the magnetic field increases the degeneracy and moves the Fermi level towards a lower level. Each time it crosses the Landau level with extended states the longitudinal resistivity is nonzero and perpendicular one jumps from one plateau to the next. The question arises. If some states get localised so why is current is the same (to give a plateau at constant voltage). The answer is provided by the compensation theorem (see [8] and references cited there) which can be proved on general grounds. In the system at hand it states that *if some states get localised the rest of them carry more current just to compensate for those which cannot.* This explains the effect of appearance of flat plateaus in R_{yx} and vanishing of R_{xx} in spite of (or in fact, rather, thanks to) impurities.

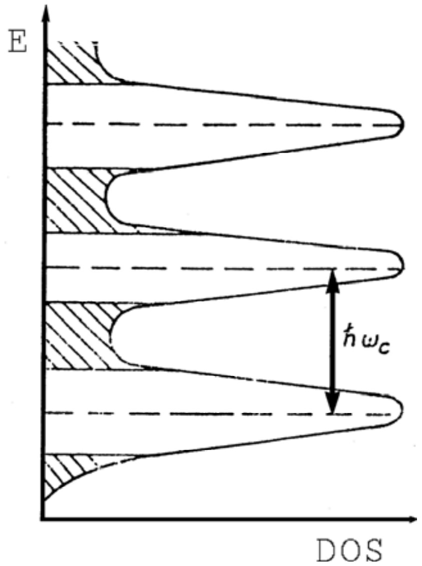

Fig. 3. Schematic dependence of the density of states on the energy for a two-dimensional electron gas subject to strong perpendicular magnetic field in the presence of impurities. Shaded regions indicate localised states. Presumably the regions of extended states are limited to the Landau level energies only (dashed lines).

3. APPLICATIONS

Here we recall the applications of the QHE in metrology as (i) a resistance standard and (ii) for precise measurements of fine structure constant and briefly describe recent proposals to use it in building quantum gates.

3.1. α and Ω

The possible application of the QHE as a resistance standard has first been recognised by Klaus von Klitzing [6]. At present the effect is used in many metrology/ standard laboratories all over the world to reproduce, calibrate and maintain the unit of resistance. The value of resistance corresponding to $\nu = 4$ in Eq. (2) has been adopted for use in laboratories starting on 1st January 1990. The quantum of resistance h/e^2 has been called the von Klitzing constant denoted by R_K and its value (as known in 1988, R_{K-90}) defined to be

$$R_{K-90} \equiv \frac{h}{e^2} = 25812.807\Omega\,. \tag{5}$$

In this context and in connection with recent proposals to simultaneously use the QHE and the Josephson effects in maintaining an indirect standard of ampere [13] it is worthwhile to recall the adopted value of the Josephson constant

$$K_{j-90} \equiv \frac{2e}{h} = 483597.9\,\mathrm{GHz\,/\,V}. \tag{6}$$

If successful, the realisation of the ampere would open a road to define a quantum standard of the mass: the kilogramme. To be precise the realisation of this proposal would require very precise measurement of the quantum resistance via QHE at the frequency of about 1 kHz, usually used in metrological experiments. The point of concern is connected with the localised states lying in-between the Landau levels. For DC current they do not contribute to dissipative transport. This is no more true for time dependent circuits. Theoretical estimations show that the departures from the quantised values at the plateaus grow with frequency ω as $\omega^{0.5}$ and restrict the accuracy of measurements to about few ppm at 1 kHz.

It has to be stressed that the adopted values of the constants (5) and (6) do not mean that the values of Planck's constant h and elementary charge e are defined. Both h and e are constant of Nature, cannot be defined and are subject to precise determination by already known and new emerging methods. It is the value of the Hall resistance at $\nu = 1$ which has been defined for the purpose of international inter laboratory comparisons. Moreover the quantum Hall resistance is much more stable than any standard wire resistance. In metrology laboratories one needs the value of 1, so a number of additional steps have to be performed to scale down the QHE resistance to this value, but these issues are outside the present discussion.

The Sommerfeld fine structure constant α is defined as $\alpha^{-1} = (2/\mu_0 c)(h/e^2)$ with c denoting the velocity of light and μ_0 the vacuum permeability. In SI both these constants are defined, so QHE directly measures the fine-structure constant. This aspect of QHE has been underlined already in the title *New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance* of the first publication announcing the discovery of QHE [4]. Again the defined value of R_{K-90} does not imply that α has been defined.

3.2. From classical to quantum PC

The devices used to study the two dimensional electron gas, namely the silicon metal-oxide-semiconductor field-effect-transistors (Si-MOSFET) and GaAs-AlGaAs heterostructures are standard building blocks of modern electronic instruments, in particular processors and other parts of computers. The two dimensional gas operating at room temperature does not show its full quantum nature as discussed above. As the classic computers are thought to be replaced by their quantum ascendants and one of the proposals is connected with QHE let us discuss some aspects of quantum information story.

The classic processors use single bits 0 and 1 and classic Boolean logic to store and process information. The quantum computers instead will make use of the quantum mechanics law which says that the linear combination $|q>$ of states, say $|0>$ and $|1>$

$$|q> = \alpha|0> + \beta|1>, \tag{7}$$

where $\alpha^2 + \beta^2 = 1$, is also a perfectly well allowed quantum state of the two state system. In quantum information theory such a state, which in a sense represents ”any state between |0 > and |1 >” is called a quantum bit or qubit. The elementary operations of qubits are called quantum gates. The number of quantum gates performing operations on qubits according to quantum logic form a quantum processor or computer. There is a number of problems to be solved before such a device could be built. They involve the questions of irreversible computations as the quantum mechanical evolution is unitary and thus reversible, the decoherence i.e. the leakage of quantum information from qubits due to their interaction with environment, the preparation of states, addressing individual qubits and reading off the answer when information processing is finished. These problems are to complicated to be discussed in detail here.

Let us only say that it has been proposed to use arrays of nuclear spins [14] embedded in the two dimensional electron gas in the regime of QHE as efficient processors immune to fast decoherence [15]. This proposal relies on the weak nuclear spin relaxation processes at low temperatures. This is due to lack of dissipation in the electron gas in regime of the quantum Hall effect. The measured relaxation times were in the range of several minutes. The gap in the spectrum slows down relaxation and decoherence.

Use of the special fractional QHE states as very efficient topologically protected qubits with estimated error rate for a logical NOT operation in the range 10^{-30} has been recently proposed [16]. The future will show whether any of these proposals could be realised.

4. QUANTUM MEASUREMENTS AND METROLOGY

The continuous trends to make electronic circuits smaller and smaller will soon reach the limits where quantum effects start to determine their functioning. There are two broad subject which should be mentioned in this context. On the one side submicron devices have been invented and studied in which a single electron can be registered. These so called single electron tunnelling structures (known as single 6 electron transistors, turnstiles and pumps) can be controlled in such a way that single electrons (or an integer number of them) can be transfered per operational cycle. They consist of the central island (dot) connected by the tunnel barriers to external electrodes. The spectrum of the small dot is quantised and its capacitance C is small. At low temperature the number of electrons on a dot is well defined, provided the coupling of it to the electrodes is weak enough. Addition of an extra electron to the island requires energy equal or bigger than the charging energy $e^2/2C$. Otherwise the tunnelling of electrons is blocked. The conditions for such Coulomb blockade are: (i) suppression of thermal fluctuations, which means that the thermal energy has to be smaller than the charging energy and (ii) suppression of quantum fluctuation, which means that the tunnelling rate should be small or resistance R of the tunnel junction large: $R \geq h/e^2$.

The circuit consisting of two islands in series connected to each other and external electrodes by tunnel barriers is known as a single electron pump. Each island can be tuned by the gate electrode. Under the right conditions an AC voltage applied between the gate electrodes will pump electrons between the source and drain leads. The resulting current I is directly related to the AC voltage frequency f via

$$I = ef. \tag{8}$$

Such pumped current has been measured experimentally with an accuracy of 1 ppm with the current in the range of nA. The device has recently been proposed as a potential current standard (see [17] and references cited there).

Quantum mechanics imposes the fundamental limits on the precision of measurements of complementary observables, such as position and momentum, different components of angular momentum, etc. These Heisenberg uncertainty limits can never be beaten. In practice there are other less fundamental limits, like thermodynamic fluctuation limit or shot noise limit. It turns out that these limits can be beaten by using the carefully designed measurements strategies [18]. The theoretically predicted enhancement of precision for N identical probes (photons, electrons, etc.) scales as $\sqrt{N}$ and may thus reach the limit $1/N$. The ways to achieve the increased accuracy is to make use of squeezed or entangled quantum mechanical states. These quantum effects allow the harnessing quantum mechanics in order to increase the precision. We call this process ”quantum metrology”.

5 SUMMARY AND CONCLUSIONS

Let us briefly summarise: the quantum Hall effect (QHE) manifests itself as a quantisation of the nondiagonal elements (ρ_{xy}) of the resistivity tensor accompanied by the simultaneous vanishing of diagonal elements ρ_{xx} of it for ranges of the magnetic field. For integer QHE $\rho_{xy} = \frac{h}{\nu e^2}$, where h is the Planck's constant, e charge of an electron and ν is an integer. There also exists a fractional quantum Hall effect for which ν is a simple fraction. It is observed at low temperatures and high magnetic fields in high mobility two dimensional electron gas. The very high accuracy of the quantisation lead to the application of the IQHE for the reproduction of the SI unit of the resistance - the ohm.

The intensive work is being done to use the IQHE together with the Josephson effect to design a quantum standard of the Ampere and as a result to replace the very classic unit of mass - the kilogramme introduced in 1889 by a quantum one.

The peculiar properties of the quantum Hall liquids make them attractive playground to propose highly speculative applications in quantum information processing. All this makes the old effect a very interesting subject of studies.

Single-electron transistors and other quantum devices are slowly making their way to applications in precise metrology as current and capacitance standards. There is no doubt they will be very useful in new metrology applications and the progress in the field is to be expected in near future.

The new quantum measurements protocols will allow to beat the shot noise limit $1/\sqrt{N}$ and obtain the ultimate quantum limit $1/N$, already experimentally achieved in quantum Hall measurements (of an electron charge [7]).

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KWANTOWE ZJAWISKO HALLA: PODSTAWY

Streszczenie

W pracy przedstawiono podstawy zjawiska i fizyczne powody pozwalające na precyzyjny pomiar oporności ρ_{xy} w warunkach całkowitego kwantowego zjawiska Halla. Omówiono zastosowanie zjawiska w metrologii jako wzorca oporności elektrycznej oraz propozycje zastosowania kwantowego zjawiska Halla do budowy qubitu - podstawowego elementu przyszłych komputerów kwantowych. Na zakończenie wykładu przedstawię kilka zagadnień związanych z zastosowaniem urządzeń jednoelektronowych w metrologii i tzw. metrologią kwantową, która w wykorzystaniu kwantowych praw szuka metod pomiaru z precyzją większą niż $1/\sqrt{N}$.